may be considered as an alternate to the approximation given in Fig. 8 of Young et al.[1]

Finally, it may be noted that by redefining d, (1) through (4) may be applied to the high-pass case. Moreover, this approach is not limited to microwave filter applications, and it seems likely that it has been used by others. However, the author has not seen it in print.

R. C. Smythe
Piezo Technology, Inc.
Orlando, Fla.

Multisection Microwave Phase-Shift Network

This correspondence extends the analysis of a phase-shift network consisting of a cascade of pairs of coupled transmissions lines connected together at their far ends[1] (see Fig. 1), to any value of n. Such cascaded all-pass networks, also known as microwave C-sections, have recently been analyzed by Steenart [1] and Zysman and Matsumoto [2]. Cristal [3] has solved the problem of analysis and exact synthesis of cascaded C-sections. Matrix methods are generally used and Richards' theorem [4] is employed, thereby restricting the various coupled sections to have equal lengths. In this analysis, the various sections may have unequal lengths.

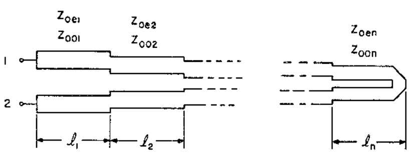

Fig. 1.

Earlier, Jones and Bolljahn [5] gave a formula for the phase shift ϕ through a single C-section terminating in $Z_0=(Z_{0o}Z_{0e})^{1/2}$, where Z_{0o} and Z_{0e} are the odd and even mode impedances, respectively, of the coupled-lines. Thus, for the single microwave C-section

$$\phi = \cos^{-1}\left(\frac{\rho - [\tan\theta]^2}{\rho + [\tan\theta]^2}\right) \qquad (1)$$

where $\rho = Z_{0e}/Z_{0o}$, and $\theta = 2\pi l/\lambda$ is the electrical length of the coupled section of physical length l, and λ is the wavelength in the medium. Dr. S. B. Cohn first suggested the use of the coupled-line all-pass network in broadband 90-degree differential phase-shift networks, which lead to a work of the writer containing the formula for the two-section ($n=2$) phase-shift network [6],

$$\phi(n=2) = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (2)$$

where

$$\theta_1' = \theta_1 + \tan^{-1}\left[(Z_{0o2}/Z_{0o1})\tan\theta_2\right].$$

Here the subscripts 1 and 2 refer to the first and second coupled sections. At the same time, the writer derived, but did not publish, the formula for the three-section coupled-line phase-shift network, which is given below

$$\phi(n=3) = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (3)$$

where

$$\theta_1' = \theta_1 + \tan^{-1}\left[(Z_{0o2}/Z_{0o1})\tan\theta_2'\right],$$

and

$$\theta_2' = \theta_2 + \tan^{-1}\left[(Z_{0o3}/Z_{0o2})\tan\theta_3\right].$$

Equations (1) to (3) now suggest the following formula for the general n-section cascade of microwave C-sections:

$$\phi_n = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (4)$$

where

$$\begin{aligned}\theta_1' &= \theta_1 + \tan^{-1}(\sigma_{12}\tan\theta_2'),\\ \theta_2' &= \theta_2 + \tan^{-1}(\sigma_{23}\tan\theta_3'),\\ &\vdots\\ \theta_{n-1}' &= \theta_{n-1} + \tan^{-1}(\sigma_{(n-1),n}\tan\theta_n'),\\ \theta_n' &= \theta_n,\end{aligned}$$

and

$$\sigma_{i,i+1} = (Z_{0ei}/Z_{0e(i+1)}) = (Z_{0o(i+1)}/Z_{0oi}).$$

The foregoing result holds only when $(Z_{0oi}Z_{0ei}) = Z_0^2$.

Equations (2) and (3) were obtained as follows. An even-mode input of $(+\frac{1}{2}, +\frac{1}{2})$ volt from a pair of Z_0 sources was assumed at Ports 1 and 2, with the far end open-circuited. Likewise, an odd-mode input of $(+\frac{1}{2}, -\frac{1}{2})$ volt was assumed at Ports 1 and 2, respectively, with a short-circuit termination at the far end. The two normal-mode inputs add to a coupled-mode input of (+1, 0) volt at Ports 1 and 2, which means that power from the generators is flowing into Port 1 but not Port 2. We now require that the output wave be $(0, e^{j\phi})$, which is the condition for a perfect match, and which also is in conformity with the principle of the conservation of energy. This condition on the output wave may also be put in terms of the even-mode and odd-mode reflection coefficients

$$(\Gamma_e + \Gamma_o) = 0 \qquad 1/2(\Gamma_e - \Gamma_o) = e^{j\phi}. \qquad (5)$$

The reflection coefficients Γ_e and Γ_o were then found with the aid of transmission-line theory, and it was readily ascertained that the desired conditions (5) could be met by letting $(Z_{0ei}Z_{0oi}) = Z_0^2$ for all i. The two expressions (5) in Γ were then combined and solved for the phase of the output wave at Port 2,

$$\phi = \cos^{-1}\left[\mathrm{Re}\,(\Gamma_e)\right] \qquad (6)$$

yielding (2) and (3).

B. M. Schiffman
Stanford Research Institute
Menlo Park, Calif.

Manuscript received December 9, 1965.

[1] The original work [6] was sponsored by the Air Force Cambridge Research Center, Bedford, Mass., under Contract AF 19(604)-1571.

References

[1] W. J. D. Steenart, "The synthesis of coupled transmission line all-pass networks in cascades of 1 to n," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-11, pp. 23–29, January 1963.
[2] G. I. Zysman and A. Matsumoto, "Properties of microwave C-sections," *IEEE Trans. on Circuit Theory*, vol. CT-12, pp. 74–82, March 1965.
[3] E. G. Cristal, "Analysis and exact synthesis of cascaded transmission line C-section all-pass networks," to be published.
[4] P. I. Richards, "Resistor transmission line circuits," *Proc. IRE*, vol. 36, pp. 217–220, February 1948.
[5] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 75–81, April 1956.
[6] B. M. Schiffman, "A new class of broadband microwave 90-degree phase shifters," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-6, pp. 232–237, April 1958.

Dual Reflex Klystron Cavity-Beam Interaction

While the occurrence of spurious resonances, or the detection of harmonic content, is not uncommon in reflex oscillators, the particular case wherein the tube performs reflex klystron functions at two different frequencies has not been reported heretofore. This phenomenon was observed in the course of a search for spurious output signals conducted on Raytheon QKK754 CW communication klystrons. X-band output was detected from these tubes using the RF equipment shown in Fig. 1.

The amplitude of the X-band signal was strongly dependent on external loading. Under optimum C-band load conditions, its power level was 25 to 27 dB lower than the C-band output of the tubes. The spurious power content rose by 6 to 8 dB when the load was adjusted for maximum output at X-band.

Upon sweeping the reflector with 60-cycle ac voltage, it was possible to monitor the $2\frac{3}{4}$ and $3\frac{3}{4}$ C-band and the $5\frac{3}{4}$, $6\frac{3}{4}$, and $7\frac{3}{4}$ X-band reflector modes simultaneously. These modes are presented on the same trace and separately in Fig. 2, to simplify viewing. The dual reflex klystron processes are evident from these oscilloscope traces.

As the plots of Fig. 3 reveal, gap tuning of the C-band resonator resulted in continuous tuning of the X-band mode over most of the mechanical tuning range of the tube. The nonintersecting reflector voltage curves provide a clear indication that there was no simultaneous X-band oscillation in the tubes at the peak of the $2\frac{3}{4}$ C-band reflector mode.

Further study of the dual cavity-beam interaction suggested that the reentrant cavity contained an interacting resonance in the first cutoff or TM_{011} coaxial mode, shown in Fig. 4, in addition to the regular TM_{010} mode.

To confirm this, the resonant frequency of the TM_{011} mode was computed along the lines developed by MacKenzie.[1] The result-

Manuscript received November 15, 1965; revised December 13, 1965.

[1] L. A. MacKenzie, "Klystron cavities for minimum spurious output power," Cornell Research Rept. EE-418, School of Electrical Engineering, Cornell Univ., Ithaca, N. Y., January 1959.

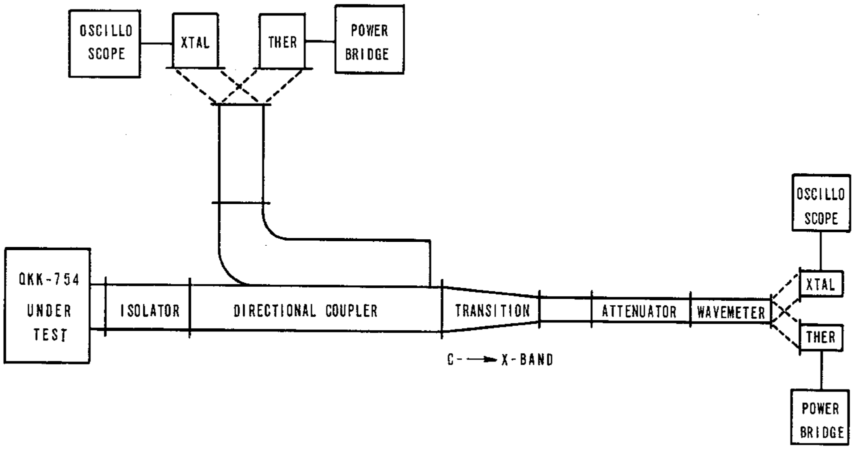

Fig. 1. Equipment used in the detection and measurement of *C*- and *X*-band power.

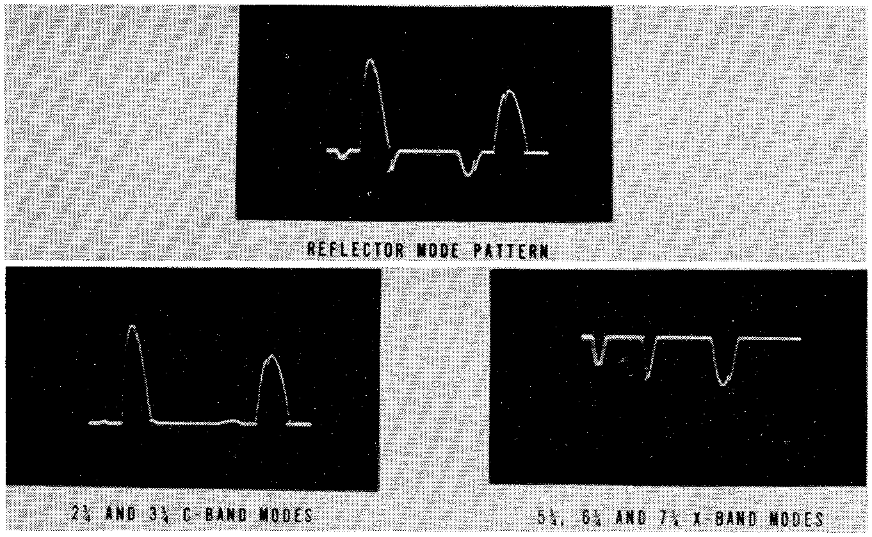

Fig. 2. Oscilloscope traces of *C*- and *X*-band reflector modes.

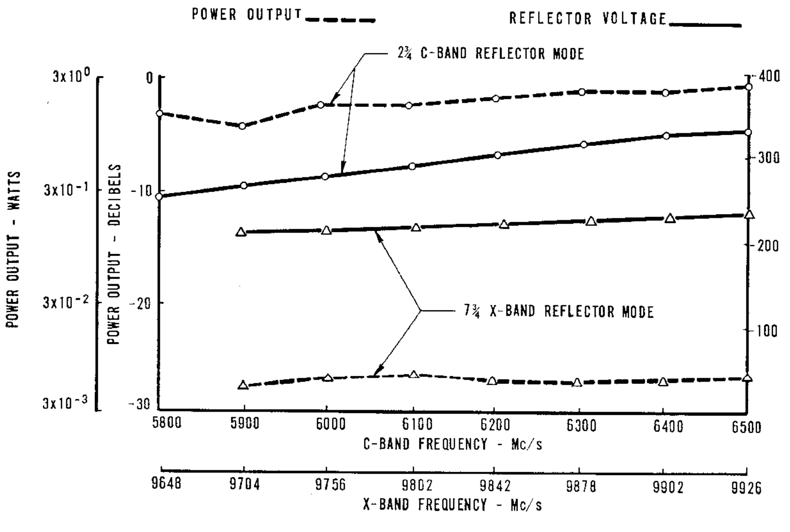

Fig. 3. Curves of power output and reflector voltage vs. frequency.

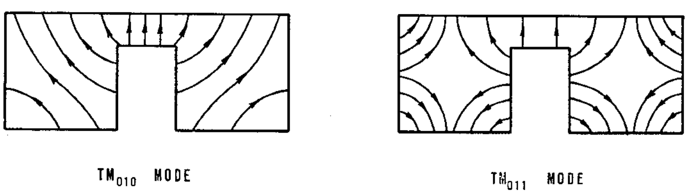

Fig. 4. Electric field configuration for beam interacting cavity modes.

ing figure of 9120 Mc/s fell within 7 percent of that of the detected output signal when the tube oscillated at a primary frequency of 6000 Mc/s.

It is worth noting that the amplitude of the secondary, higher frequency oscillation encountered in the above case may be amplified or suppressed through proper modification of the dimensional parameters of the resonator. The former has special significance for the millimeter region since it renders possible the operation of a reflex klystron at a frequency considerably higher than that which the physical dimensions of its circuit and beam would normally allow.

ACKNOWLEDGMENT

Thanks are due to R. W. Nersesian who performed most of the measurements reported in this correspondence.

J. J. HAMILTON
Beam Tube Group
Raytheon Company
Waltham, Mass.

An Exact Analysis of Varactor Frequency Multipliers

A novel yet simple approach to the exact analysis of an abrupt-junction frequency doubler is presented, utilizing the fact that the voltage is proportional to the square of the charge. Penfield and Rafuse[1] were the first to consider the problem in an exact analysis. By imposing certain constraints they obtained useful design information with the aid of a large-scale computer. Through different constraints, the present analysis also offers an exact analysis, but the solution is expressed in a closed form. The series model of incremental elastance $S(t)$ and resistance R_s is shown in the circuit of Fig. 1.

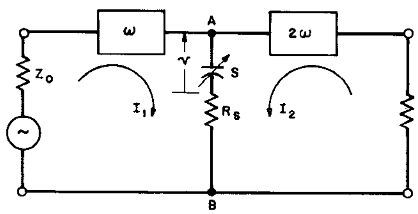

Fig. 1. Doubler circuit model.

The total current i and charge q flowing through the varactor diode are, respectively,

$$i = i_1 + i_2 = I_1 \cos \omega t + I_2 \cos (2\omega t + \theta) \quad (1)$$

and

$$q = \frac{I_1}{\omega} \sin \omega t + \frac{I_2}{2\omega} \sin (2\omega t + \theta) + K, \quad (2)$$

where θ is the phase angle between the fundamental and second harmonic in second harmonic time measure, and K is the average charge to be determined by the bound-

Manuscript received May 5, 1965; revised December 13, 1965.

[1] P. Penfield and R. P. Rafuse, *Varactor Applications*. Cambridge, Mass.: M.I.T. Press, 1962.